





NPS-53-87-003

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STATISTICAL INTERPOLATION BY ITERATION

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Technical Report for Period

April 1987-June 1987

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Frepared for: Office of Naval Research

Applied Research and Technology

Directorate

Arlington, VA 22217-5000

87 8 21 075

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SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION	READ INSTRUCTIONS BEFORE COMPLETING FORM			
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER		
NPS-53-87-003	A146.0023			
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED		
Statistical Interpolation by Iteration		Technical Report		
		6. PERFORMING ORG. REPORT NUMBER		
		4/87 - 6/87		
7. AUTHOR(#)		8. CONTRACT OR GRANT NUMBER(#)		
Richard Franke				
3. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS		
Naval Postgraduate School		Program Element 61153N		
Monterey, CA 93943		N0001487WR22049		
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE		
Office of Naval Research		April 1987		
Arlington, VA 22217-5000		13. NUMBER OF PAGES		
14. MONITORING AGENCY NAME & ADDRESS(II dilleren	I from Controlling Office)	18. SECURITY CLASS. (of this report)		
		Unclassified		
		15a DECLASSIFICATION/DOWSGRADING		
		15a. DECLASSIFICATION/ DOWN GRADING SCHEDULE		
Approved for public release;	distribution	unlimited		
17. DISTRIBUTION STATEMENT (of the abetract entered i	in Block 20, if different from	n Report)		
18. SUPPLEMENTARY NOTES				
19. KEY WORDS (Continue on reverse elde if necessary and identify by block number)				
objective analysis, interpolation, statistical interpolation, optimum interpolation, successive corrections				
20. ABSTRACT (Continue on reverse side if necessary and	identify by block number)			
An iterative (successive correction) method for objective analysis due to Bratseth is considered. The method converges to the statistical interpolation result in the limit. The properties of the scheme and a variation of it are discussed, and the results of some simulations performed earlier for other methods are given and compared.				

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1.0 INTRODUCTION

At the urging of Jean Thiebaux and David Parrish of the National Meteorological Center, I recently extended some simulations reported on previously (see Franke, 1985) to include an idea advanced by Bratseth (1986). The scheme proposed there is an iterative scheme for objective analysis, similar to successive correction methods (SCM) (see Cressman (1959)), with the weights chosen in such a way that the iteration converges to the results of a statistical interpolation (SI) scheme (see Gandin (1963)). This would appear to be attractive because of the ease of applying SCM and the skill of SI. The question that remains is whether or not the proposed scheme converges rapidly enough in a practical setting to justify its use. The simulations described in this report were close to realistic. Further investigation of properties of the scheme give information about why the method performs as it does in the simulations and whether similar results can be expected in practice.

The second section reviews the ideas of Bratseth, and discusses a generalization of the scheme within the context of iterative methods for solving linear systems. In the third section the results of the simulations corresponding to those performed in Franke (1985) are presented with some conclusions. Section 4 gives suggestions for further investigation.

2.0 THE BRATSETH METHOD

The basis for the method flows from the following ideas. Let observation points \mathbf{x}^i , $i=1,\ldots,n$ be given, and let $\mathbf{c}_{i,j}$ represent the spatial covariance between the background plus observa-

tion errors at points x^i and x^j , while c_{ix} represents the covariance between the background errors at x^i and x, the latter an arbitrary point at which the background error is to be estimated. Then the weights p_j , $j=1,\ldots,n$, for the SI correction at point x are the solution of the equations

(1)
$$\sum_{j=1}^{n} c_{ij} p_{j} = c_{ix}, i=1,...,n$$
,

with the analyzed values being given by

(1a)
$$F_{x}^{A} = F_{x}^{P} + \sum_{j=1}^{n} p_{j} (F^{O} - F_{j}^{P})$$
.

Here F_i^A , F_i^F , and F_i^O represent the analyzed, predicted (background), and observed values at point x^i , respectively, while F_x^A and F_x^F represent analyzed and predicted values at x.

Using SCM with the spatial covariance function as the weight function yields the iteration

(2)
$$F_{x}^{A}(k+1) = F_{x}^{A}(k) + \sum_{j=1}^{n} a_{x,j}(F_{j}^{D}(k) - F_{j}^{A}(k))$$
,

where $a_{\rm xj}$ is $c_{\rm xj}/M_{\rm xj}$ for some normalizing factor $M_{\rm xj},$ which is typically taken to be

Bratseth's observation is: if equation (2) is used to evaluate the $F_j^A(k)$, instead of interpolating from the grid, then the iteration given by (2) will converge (when it converges) to the solution of (1), provided the $M_{\chi j}$'s are chosen independent of χ . This observation (in a limited sense), was also made by Franke and Gordon (1983), where the $M_{\chi j}$'s were all taken to be the same.

The key to analyzing the behavior of the iteration lies in

the iteration at the observation points, since the grid point values (while being the real interest) have no effect on convergence. In matrix form, the iteration for analyzed values at the observation points has the form

(4) $F^A(k+1) = F^A(k) + A(F^O(k) - F^A(k))$, where A is the matrix (a_{ij}/M_{ij}) , F^O is the vector of observed values and $F^A(k)$ is the vector of analyzed values $(k^{th}$ iteration) at the observation points. The predicted values at the observation points (obtained by interpolation from the grid), F^P , are used as the initial iterate, $F^A(0)$. Since the values of $M_{\chi j}$ are to be chosen independent of χ , denote them by M_j . Then A is of the form A = CM, where C = (c_{ij}) and M = diag (M_i) .

Bratseth suggests (in our context of independent observation errors)

$$M_{i} = \sum_{i=1}^{n} |c_{ij}|.$$

The effect of this set of M_j's will be to ensure that the matrix has all (absolute) column sums equal to one. This shows that all eigenvalues of A are bounded by one, and since A is positive definite, all eigenvalues are between zero and one. The iteration matrix for the scheme is I-A, which is then seen to have all eigenvalues between zero and one, as well, ensuring convergence of the scheme. The rate of convergence is proportional to the largest eigenvalue of I-A, therefore when A has small eigenvalues the convergence is slow.

Replacing the matrix A by A/ α (equivalently, replacing the M_J by α M_J) will result in a convergent scheme provided α lies within certain limits as noted by Bratseth in another context,

 $0<\alpha^{-1}<2$. Let λ_j represent the eigenvalues of A, in decreasing order. Then, values of α greater than one will cause slow convergence since the largest eigenvalue of I-A/ α will be $1-\lambda_n/\alpha>1-\lambda_n$. Thus, to minimize the largest eigenvalue of I-A/ α , one should take α to satisfy $\alpha=(\lambda_n+\lambda_1)/2$. Because of the unknown properties of the associated eigenvectors relative to the error in the initial iterate, such a value will probably not be optimum for a scheme which does not iterate to convergence. In any case, computation of λ_1 and λ_n is not feasible in practice. I also note that as a parameter, α^{-1} behaves much the same way as an over-relaxation factor such as used in Gauss-Seidel and other iterative schemes for linear systems of equations.

3.0 SIMULATION RESULTS

The basic simulations performed for a variety of objective analysis schemes in Franke (1985) (see that paper for details of the simulations) were conducted for several variations of the Bratseth method. Initial guesses at the analyzed values were obtained by piecewise cubic interpolation of predicted values from the grid to the observation points. The parameters for the simulations were: 500 mb height field (see Koehler, 1979), standard deviations of the error in the predicted and observed values are 30 m and 10 m, respectively, and the assumed (and true) correlation function for the predicted error was the isotropic negative squared exponential, $\exp(-(d/10)^2)$, where d is distance in degrees. The results for the Bratseth scheme using 3, 5, and 10 iterations, with $\alpha = 1.0$, 0.75, 0.65, and 0.55 are given in Table 1.

No. It.	α = 1.0	α = 0.75	α = 0.65	$\alpha = 0.55$
3	10.48	9.34	9.27	12.75
	10.24(2.27)	9.13(1.97)	9.05(2.02)	11.91(4.57)
5	8.82	8.04	7.77	9.80
	8.63(1.80)	7.89(1.59)	7.62(1.53)	9.33(2.94)
10	7.33	6.91	6.75	7.02
	7.20(1.40)	6.79(1.31)	6.63(1.28)	6.88(1.39)
(6.09	6.09	6.09	6.09
	5.98(1.19)	5.98(1.19)	5.98(1.19)	5.98(1.19)

Table 1: RMS analysis errors for Bratseth's scheme. This table corresponds to entries in Table 2, PW cubic column in Franke (1985). Entries are: RMS analysis error Mean RMS error (Std. Dev.)

The table shows that (in this context) the scheme is less skillful than Barnes' scheme for three iterations, while additional iterations and smaller values of α yield a scheme which is more skillful than Barnes' scheme. If the iterations are continued, the scheme converges to SI (OI here, since the actual statistical properties have been assumed), however it is doubtful that more than 10 iterations would be cost effective in practice. Use of smaller values of α are seen to be quite useful for the early iterations. In this case the smallest eigenvalue of A is $\lambda_{\rm n} < 0.01$, whereas $\lambda_{\rm 1} = 1$, which indicates the optimum value of α is close to 0.5 . However, RMS errors for a few iterations tend to be larger when $\alpha = 0.55$, probably because the decomposition of the error in terms of eigenvectors results in larger components corresponding to λ_1 , which is slowly damped if α is near its optimum value. Thus, the optimum value of α for a given number of iterations is somewhere between the theoretical optimum and $\alpha=1$. Note that for 3 iterations, the RMS errors with α = 0.75 and α = 0.65 are nearly the same, while for α = 0.55 the error is larger. As the iteration count increases, faster convergence rates occur for the smaller values of α .

Table 2 shows something of the the sensitivity of the scheme to misspecification of the ratio between prediction and observation errors. The set of realizations was different here than for those that made up Table 1, which accounts for column 1 of Table 2 differing from column 3 of Table 1. The rate of convergence seems to be improved slightly here, although the performance of SI is deteriorated.

No. It.	α = 0.65	rglie=20	rolie=5
3	9.88	9.59	9.69
	9.67(2.05)	9.34(2.21)	9.50(1.93)
5	8.37	8.26	8.21
	8.21(1.65)	8.05(1.86)	8.04(1.67)
10	7.30	7.2 4	7.12
	7.16(1.45)	7.07(1.57)	6.97(1.44)
∞ (=>SI)	6.40	6.47	6.88
	6.27(1.30)	6.32(1.35)	6.74(1.38)

Table 2: RMS analysis errors for Bratseth's scheme. This table corresponds in part to Figure 13, Franke (1985). Nominal values of r and r were 30 m and 10 m, respectively, while the analysis was given 20 m and 5 m, respectively for columns 2 and 3. Entries are: RMS analysis error Mean RMS error (Std. Dev.).

Based on these simulations, it is not clear that the method is superior to a highly tuned version of Barnes' scheme, for a reasonable number of iterations. Sensitivity to misspecification of the correlation function was not investigated, but this can be expected to be similar to that of statistical interpolation since the scheme converges to the SI approximation.

4.0 FURTHER THOUGHTS

The analysis of iterative methods for linear systems reveals that the components of the error vector corresponding to large eigenvalues of the iteration matrix are most slowly damped. The large eigenvalues of $I-A/\alpha$ correspond to the small eigenvalues of A in the present discussion. The eigenvectors corresponding to small eigenvalues tend to have "spikes" at a few of the observation points. Because of this it is possible that the errors left after a finite number of iterations correspond to components which will be damped out during the initialization phase, prior to beginning the numerical integration of the dynamical equations in NWF. Thus, it is possible that in a practical setting the performance of the Bratseth scheme may be much better than indicated by the raw RMS errors shown in Table Whether or not this is the case will probably be quite difficult to determine without commitment of significant resources to conduct full scale verification runs.

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5.0 ACKNOWLEDGEMENTS

This small investigation was prompted by Jean Thiebaux and David Parrish of the National Meteorological Center. It was supported by the Office of Naval Research under Program Element 61153N, Project No. BR033-02-WH.

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